# Particle transfer and fusion cross-section for Super-heavy nuclei in dinuclear system

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## Abstract

Within the dinuclear system (DNS) conception, instead of solving Fokker-Planck Equation (FPE) analytically, the Master equation is solved numerically to calculate the fusion probability of super-heavy nuclei, so that the harmonic oscillator approximation to the potential energy of the DNS is avoided. The relative motion concerning the energy, the angular momentum, and the fragment deformation relaxations is explicitly treated to couple with the diffusion process, so that the nucleon transition probabilities, which are derived microscopically, are time-dependent. Comparing with the analytical solution of FPE, our results preserve more dynamical effects. The calculated evaporation residue cross sections for one-neutron emission channel of Pb-based reactions are basically in agreement with the known experimental data within one order of magnitude.

**keywords:** super heavy-nuclei; dinuclear system; driving potential; master equation; complete fusion

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## I. INTRODUCTION

Since the nuclear shell model based on the Strutinsky shell-correction method predicted that the next doubly magic shell closure beyond $^{208}$Pb is at a proton number between Z=114 and 126 and a neutron number N=184 [1, 2, 3, 4, 5], a super-heavy nuclear island of stability is expected, and the outstanding aim of experimental investigation is the exploration of this region of super-heavy elements. Up to now 16 new elements beyond fermium (charge number Z=100) have been synthesized in the world, but where is the center of the island is still an open question [6, 7, 8, 9]. Furthermore, super-heavy elements are extremely difficult to be synthesized because the formation cross sections are very small, and the excitation functions are very narrow. So a better understanding of the physics conception on the super-heavy nucleus is very important. Several theoretical models have been developed to describe the reaction dynamical mechanism [10, 11, 12, 13, 14, 15]. Among these models, Adamian et al. have investigated the reaction mechanism of the super-heavy element (SHE) formation in the concept of a dinuclear system (DNS). In this model the formation is discussed as a competition between quasi-fission and complete fusion, and the cross sections are calculated including nuclear structure effects. The model not only reproduces the experimental data quite well, but also predicts the optimal projectile-target combination as well as the optimal bombarding energy to form a certain SHE. It is shown that the DNS model is a powerful tool to describe SHE production, and is one of a few models so far which gives no contradiction to available experimental data.

In the DNS model by Adamian et al. [14, 15, 16, 17, 18] it is considered that after full dissipation of the collision kinetic energy, a DNS is formed. The DNS evolves to a compound nucleus by nucleon transfer from a light nucleus to a heavy one, and the Fokker-Planck-Equation(FPE) is used to describe the diffusion process. However, in this work [15, 16, 17, 18] a Gaussian-type function solution of FPE is adopted with a harmonic oscillator approximation of the potential energy surface. Or Adamian et al. have used the Kramers-type expression solution under the quasi-stationary approximation and the harmonic oscillator approximation as well. In fact the potential energy surface of DNS including shell structure and even-odd effect corrections deviates much from the harmonic oscillator form and the approximation to it weakens the structure effect. Recently they have solved Master equation numerically for calculating the charge, mass, and kinetic energy distribu-

tions of quasifission products, describing the evolution of a dinuclear system in charge and mass asymmetries and the decay of this system along the internuclear distance. But the coupling of the diffusion process with relative motion is not considered [19, 20]. The dissipation of relative kinetic energy of heavy ions should be a function of the interaction time of the DNS [21]; during this time the nucleon transfer is coupled with the energy dissipation, which is coupled with the angular momentum and the dynamical deformation relaxations as well. To take these effects into account within the DNS concept, instead of solving FPE analytically, we have solved the Master equation numerically to treat the nucleon transfer, so that the harmonic oscillator approximation to the driving potential is avoided, and the nucleon transfer process is coupled with the relative motion [22]. Based on the fusion probability obtained from the numerical solution of the Master equation, together with the calculations of the survival probabilities, the evaporation residue cross sections are obtained.

In section II the Master equation is introduced to calculate the fusion probability of the DNS. The nucleon transition probability, the rate of the quasi fission decay probability, the local excitation energy and the driving potential of the system are explained. In section III, for Pb-based one-neutron emission cold fusion reactions and for compound nuclei from Z=106 to 118 (only for even Z), the driving potentials of the DNS, the optimal excitation energies, the fusion probabilities, the survival probabilities, as well as the evaporation residue cross sections are calculated. Our summary is given in section IV.

## II. NUCLEON TRANSFER IN DNS CONCEPT

### A. The evaporation residue cross section in DNS concept

In the DNS concept the evaporation residue cross section can be written as a sum over all partial waves J [16]

$$\sigma_{ER}(E_{cm}) = \sum_{J=0}^{J=J_f} \sigma_c(E_{cm}, J) P_{CN}(E_{cm}, J) W_{sur}(E_{cm}, J), \tag{1}$$

where the partial capture cross section for the transition of the colliding nuclei over the entrance barrier with the penetration coefficient $T(E_{cm}, J)$ at the incident energy of center of mass $E_{cm}$ to form the DNS is given by

$$\sigma_c(E_{cm}, J) = \pi \lambda\!\!\!^{-2} (2J+1) T(E_{cm}, J), \tag{2}$$

where $\lambda\!\!\!^{-}$ is the reduced de Broglie wavelength, $\lambda\!\!\!^{-2} = \hbar^2/(2\mu E_{cm})$, with $\mu$ the reduced mass. $J_f$ is taken as the value at which the contribution of the corresponding partial wave to $\sigma_{ER}(E_{cm})$ becomes 0.01 times smaller than the contribution of $J = 0$ partial wave. For cold fusion reactions leading to super-heavy nuclei, the values of $J_f \simeq 20 - 30\hbar$. And $T(E_{cm}, J) \simeq 0.5$ are chosen for energies $E_{cm}$ near above the Coulomb barrier. The probability $P_{CN}(E_{cm}, J)$ of the complete fusion is evaluated by considering the fusion process as a diffusion of DNS in the mass asymmetry $\eta = (A_1 - A_2)/A$, with $A_1, A_2$ the mass numbers of the DNS nuclei, and $A = A_1 + A_2$. The nucleon transfer is coupled with the dissipation of the relative kinetic energy and the angular momentum, and the relaxation of colliding nuclear deformations. The survival probability $W_{sur}(E_{cm}, J)$ estimates the competition between fission and neutron evaporation of the excited compound nucleus by the statistical model and decreases much with increasing $J$, which determines the maximal contributing $J_f$.

### B. The Master equation

In the DNS model [15, 17, 18] the dynamics has been treated as a diffusion in mass asymmetry at the touching point to the compound nucleus, and in the variable R of the relative distance between the centers of the interacting nuclei, which may lead to the quasi-fission. The analytical solution of the Fokker-Planck equation [15, 17] or the numerical solution of the Master equation [20, 22] are used to describe the diffusion process. The nucleon transfer from the light nucleus to the heavy one can be described by transport theory which has been proved to be a successful tool for investigating nucleon transfer in deeply inelastic collisions of heavy ions [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33].

To solve FPE is convenient if it can be solved analytically. But in this case the potential energy surface of DNS must be linear or quadratic function of the relevant collective variables chosen to treat the fusion process, and the harmonic oscillator approximation of the potential energy surface is inevitable. To avoid this approximation the Master equation is solved numerically in order to treat the nucleon transfer in the present work. Furthermore, in our investigation the nucleon transfer is coupled with the relative motion and is considered as a time-dependent process. The evolution of the DNS is not only a diffusion process in the mass asymmetry at the touching point to the compound nucleus, but also simultaneously in the variable R of the relative distance between the centers of the interacting nuclei by

decay into the direction of increasing R, which may lead to the quasi-fission of the DNS. The fraction of the probability, which goes to quasi fission, leaks out of the evolution system, so the decay in R affects the motion of the system in $\eta$. Let $P(A_1, E_1, t)$ be the distribution function to find $A_1$ nucleons with excitation energy $E_1$ in fragment 1 at time t, where $E_1$ is not considered as an independent variable but a parameter supplied by the initial relative motion. $P(A_1, E_1, t)$ obeys the following Master Equation(ME):

$$\frac{dP(A_1, E_1, t)}{dt} = \sum_{A_1^{'}} W_{A_1, A_1^{'}}[d_{A_1} P(A_1^{'}, E_1^{'}, t) - d_{A_1^{'}} P(A_1, E_1, t)] - \Lambda^{qf}_{A_1, E_1, t}(\Theta) P(A_1, E_1, t), \quad (3)$$

where $W_{A_1, A_1^{'}} = W_{A_1^{'}, A_1}$is the mean transition probability from a channel $(A_1, E_1)$ to $(A_1^{'}, E_1^{'})$, $d_{A_1}$ denotes the microscopic dimension for the corresponding macroscopic variables. The coefficient $\Lambda^{qf}_{A_1, E_1, t}(\Theta)$ is the rate of decay probability in R, and will be described later. The sum is taken over all possible mass numbers that fragment 1 may take(from 0 to $A = A_1 + A_2$). The motion of the nucleons in the interacting nuclei is considered to be described by the single-particle Hamiltonian [23, 34]

$$H(t) = H_0(t) + V(t) \quad (4)$$

with

$$H_0(t) = \sum_k \sum_{\nu_k} \varepsilon_{\nu_k}(t) a^{\dagger}_{\nu_k}(t) a_{\nu_k}(t), \quad (5)$$

$$V(t) = \sum_{k,k^{'}} \sum_{\alpha_k, \beta_{k^{'}}} u_{\alpha_k \beta_{k^{'}}}(t) a^{\dagger}_{\alpha_k}(t) a_{\beta_{k^{'}}}(t) = \sum_{k,k^{'}} V_{k,k^{'}}(t), \ k, k^{'} = 1, 2 \quad (6)$$

The quantities $\varepsilon_\nu(t)$ and $u_{\nu\mu}(t)$ denote the single-particle energies and the interaction matrix elements, respectively. The single-particle states are defined with respect to the moving centers of nuclei and are assumed to be orthogonalized in the overlap region. Therefore, the annihilation and creation operators depend on time. The single-particle interaction matrix element is parameterized by

$$u_{\alpha_k, \beta_{k^{'}}}(t) = U_{kk^{'}}(t)\{exp[-\frac{1}{2}(\frac{\varepsilon_{\alpha_k}(t) - \varepsilon_{\beta_{k^{'}}}(t)}{\triangle_{kk^{'}}(t)})^2] - \delta_{\alpha_k, \beta_{k^{'}}}\}, \quad (7)$$

which contain five independent parameters. These are the strength parameters $U_{11}(t)$ and $U_{22}(t)$ for exciting a nucleon in fragment 1 and 2, respectively, and $U_{12}(t) = U_{21}(t)$ for

transferring a nucleon between the fragments, and the corresponding width parameters $\Delta_{11}(t) = \Delta_{22}(t)$ and $\Delta_{12}(t) = \Delta_{21}(t)$. The strength parameters are taken as($g_k = A_k/12$):

$$U_{kk'} = \frac{g_1^{\frac{1}{3}} g_2^{\frac{1}{3}}}{g_1^{\frac{1}{3}} + g_2^{\frac{1}{3}}} \cdot \frac{1}{g_k^{\frac{1}{3}} g_{k'}^{\frac{1}{3}}} \cdot 2\gamma_{kk'} \tag{8}$$

In our calculation $\Delta_{11}(t) = \Delta_{12}(t) = 2MeV$, and the dimensionless strength parameters $\gamma_{11} = \gamma_{22} = \gamma_{12} = \gamma_{21} = 3$ are taken [23, 34]. The transition probability reads:

$$W_{A_1,A_1^{'}} = \frac{\tau_{mem}(A_1, E_1, A_1^{'}, E_1^{'})}{\hbar^2 d_{A_1} d_{A_1^{'}}} \sum_{ii'} |\langle A_1^{'}, E_1^{'}, i^{'}|V|A_1, E_1, i\rangle|^2, \tag{9}$$

where i denotes all remaining quantum numbers. The memory time is:

$$\tau_{mem}(A_1, E_1; A_1^{'}, E_1^{'}) = (2\pi)^{1/2}\hbar\{\langle V^2(t)\rangle_{\ A_1,E_1} + \langle V^2(t)\rangle_{\ A_1^{'},E_1^{'}}\}^{-1/2}, \tag{10}$$

where $\langle\ \rangle_{A_1,E_1}$ stands for the average expectation value with $A_1, E_1$ fixed. Due to the dissipated energy in the evolution process of the relative motion, the nuclei are excited gradually. The excitation energy opens a valence space of width $\Delta\varepsilon_k$ in the fragment k which lies symmetrically around the Fermi energy surface. Only those particles in the states within the valence space are active for excitation and transfer. The averages in Eqs.(9) and (10) are performed in the valence space:

$$\Delta\varepsilon_k = \sqrt{\frac{4\varepsilon_k^*}{g_k}}, \varepsilon_k^* = \varepsilon^* \frac{A_k}{A}, g_k = \frac{A_k}{12}, \tag{11}$$

where $\varepsilon^*$ stands for the local excitation energy of the system, and will be given below. There are $N_k = g_k\Delta\varepsilon_k$ valence states and $m_k = N_k/2$ valence nucleons in $\Delta\varepsilon_k$. The dimension is $d(m_1, m_2) = \begin{pmatrix} N_1 \\ m_1 \end{pmatrix} \begin{pmatrix} N_2 \\ m_2 \end{pmatrix}$. The transitions for a proton or neutron are not distinguished in the transition probability since the ME is essentially restricted to one dimension. It is, however, remedied by including the explicit proton and neutron numbers of the isotopic composition of the nuclei forming the DNS in the driving potential. The averages in Eqs. (9) and (10) are carried out by using the method of spectral distributions [35, 36, 37]. We obtain

$$\langle V_{kk'} V_{kk'}^{\dagger}\rangle = \frac{1}{4} U_{kk'}^2 g_k g_{k'} \Delta_{kk'} \Delta\varepsilon_k \Delta\varepsilon_{k'} [\Delta_{kk'}^2 + \frac{1}{6}(\Delta\varepsilon_k^2 + \Delta\varepsilon_{k'}^2)]^{-1/2} \tag{12}$$

and

$$\tau_{mem}(A_k, E_k, t) = \hbar[2\pi/\sum_{kk'}\langle V_{kk'}V_{kk'}^{\dagger}\rangle]^{1/2} \tag{13}$$

According to Eq. (6) the transition probability of Eq. (9) can be written as:

$$\begin{aligned} W_{A_1,A_1'}(A_1, E_1; A_1', E_1') = \frac{\tau_{mem}(A_1,E_1;A_1',E_1')}{\hbar^2 d_{A_1} d_{A_1'}}\{[\omega_{11}(A_1, E_1, E_1') + \omega_{22}(A_1, E_1, E_1')]\delta_{A_1',A_1} \\ +\omega_{12}(A_1, E_1, E_1')\delta_{A_1',A_1-1} + \omega_{21}(A_1, E_1, E_1')\delta_{A_1',A_1+1}\}, \end{aligned} \tag{14}$$

where

$$\omega_{kk'}(A_1, E_1, E_1') = \sum_{k,k',A_1'} |\langle A_1, E_1, k|V_{kk'}|A_1', E_1', k'\rangle|^2 = d_{A_1}\langle V_{kk'}V_{kk'}^{\dagger}\rangle. \tag{15}$$

### C. The local excitation energy and the driving potential of the system

The local excitation energy is defined as the following:

$$\varepsilon^* = E - U(A_1, A_2) - \frac{(J-M)^2}{2\mathcal{J}_{rel}} - \frac{M^2}{2\mathcal{J}_{int}}, \tag{16}$$

where $E$ is the intrinsic excitation energy of the composite system converted from the relative kinetic energy loss. $M$ denotes the corresponding intrinsic spin due to the relative angular momentum dissipation and $\mathcal{J}_{int}$ the corresponding moment of inertia of the system. $J$ and $\mathcal{J}_{rel}$ are the relative angular momentum and the relative moment of inertia of the DNS, respectively. The quantities $E$, $M$, $\mathcal{J}_{int}$, $\mathcal{J}_{rel}$ calculated for each initial relative angular momentum $J$, are coupled each other due to the fragment deformation relaxation and are functions of the evolution time t.

The driving potential energy for the nucleon transfer of the DNS is:

$$U(A_1, A_2) = B(A_1) + B(A_2) - B(A) + U_C(A_1, A_2) + U_N(A_1, A_2), \tag{17}$$

where $B(A_1)$, $B(A_2)$, and $B(A)$ are the binding energies of the fragments and compound nucleus, respectively, and are taken from Ref. [39], so that the shell and paring corrections are included in them. The nuclear interaction energy can be parameterized by the Morse

potential as in Ref. [15]

$$U_N(A_1, A_2) = D(exp[-2\alpha\frac{R-R_0}{R_0}] - 2exp[-\alpha\frac{R-R_0}{R_0}]), \tag{18}$$

where $D = 2\pi a_1 a_2 R_{12}(10.96 - 0.8R_{12})$ (in MeV), $R_0 = R_1 + R_2$, and $\alpha = 11.47 + 2.069R_{12} - 17.32a_1a_2$ (dimensionless) are the depth, minimum position, and inverse width of the potential, respectively, $R_{12} = R_1R_2/R_0$ ($R_1$,$R_2$ are the radii of the nuclei). $a_1, a_2 \approx 0.54 - 0.59$. $R$ is the distance between the centers of nuclei. It is not taken as an independent variable in our calculation, but as $R = R_1 + R_2 + R_d$, where $R_d$ is chosen as the value which gives the minimum value of $U_C(A_1, A_2) + U_N(A_1, A_2)$. If the ground state deformations of the two touching nuclei are taken into account, the Coulomb interaction of the deformed DNS, $U_C(A_1, A_2)$, must be calculated numerically. For the nuclear part of the potential, in Eq. (18), nuclei are assumed as spherical but shifted to a smaller relative distance determined by the same distance between the nuclear surfaces as the one which the deformed nuclei have. In this manner the deformation of the nuclei was simulated. In principle, the deformed nuclei can have different relative orientations. Some averaging over the orientations of the nuclei has to be carried out in the initial DNS, however, the orientation which gives rise to the minimum interaction energy is in favor of the nucleon transfer. So the pole to pole orientation is chosen as the case which gives rise to the minimum energy.

The evolution of the DNS in the variable R of the relative distance between the centers of the interacting nuclei will lead to the quasi-fission of the DNS. For a given mass asymmetry $\eta$, the nucleus-nucleus interaction potential as a function of R is:

$$V(A_1, A_2, R) = U_C(A_1, A_2, R) + U_N(A_1, A_2, R) + U_{rot}(A_1, A_2, R), \tag{19}$$

where the Coulomb interaction $U_C$ is calculated numerically and the nuclear interaction $U_N$ is calculated by Eq.(18) as a function of R at each combination of the DNS. $U_{rot}$ is the centrifugal potential. The nucleus-nucleus interaction potential $V(A_1, A_2, R)$ has a pocket as a function of the relative distance R with a small depth which results from the attractive nuclear and repulsive Coulomb interactions. The probability $P(A_1, E_1, t)$ distributed in the pocket will have the chance to decay out of the pocket with a decay rate $\Lambda^{qf}_{A_1,E_1,t}(\Theta)$ in Eq.(3), which can be treated with the one dimensional Kramers rate as in Ref. [19]:

$$\Lambda^{qf}_{A_1,E_1,t}(\Theta) = \frac{\omega}{2\pi\omega^{B_{qf}}}(\sqrt{(\frac{\Gamma}{2\hbar})^2 + (\omega^{B_{qf}})^2} - \frac{\Gamma}{2\hbar})exp(-\frac{B_{qf}(A_1)}{\Theta(A_1, E_1, t)}), \tag{20}$$

which exponentially depends on the quasi fission barrier $B_{qf}(A_1)$ for a given mass asymmetry $\eta$, and the $B_{qf}(A_1)$ measures the depth of this pocket. The temperature $\Theta(A_1, E_1, t)$ is calculated by using the Fermi-gas expression $\Theta = \sqrt{\frac{\varepsilon_1^*}{a}}$ with the excitation energy $\varepsilon_1^*$ given in Eq.(11), and $a = \frac{A}{12} MeV^{-1}$. $\omega^{B_{qf}}$ in Eq.(20) is the frequency of the inverted harmonic oscillator approximating the potential V in R around the top of the quasi fission barrier. And $\omega$ is the frequency of the harmonic oscillator approximating the potential in R at the bottom of the pocket. They are determined by the local oscillator approximation of the nucleus-nucleus potential energy. The quantity $\Gamma$ denotes a double average width of the contributing single-particle states, which determines the friction coefficients: $\gamma_{ii'} = \frac{\Gamma}{\hbar}\mu_{ii'}^{-1}$, with $\mu_{ii'}$ the mass parameters. And $\Gamma \approx 2MeV$. From our calculation, the extracted average values: $\hbar\omega^{B_{qf}} \sim 2.0MeV$, and $\hbar\omega \sim 4.0MeV$.

### D. The numerical procedure

The distribution function $P(A_1, E_1, t)$ is calculated by solving Eq.(3) numerically. During the nucleon transfer process it is assumed that only one nucleon exchange is preferential. Two or more than two nucleon exchange processes at one time are negligible. These allow us to make a simplification in Eq.(3) about the transition probability that $W_{A_1,A_1'}$ are sharply and symmetrically peaked at $A_1$ and only $W_{A_1,(A_1-1)}$ and $W_{A_1,(A_1+1)}$ are significant. The consequence is that only two terms, namely $A_1^{'} = A_1 \pm 1$ remain in the summation of Eq.(3), so that the difference equations corresponding to Eq.(3) become tri-diagonal coupled algebraic equations.

The boundary of the distribution function $P(A_1, E_1, t)$ is assumed as: $P(A_1 < 0, E_1, t) = 0$, and $P(A_1 > (A_P + A_T), E_1, t) = 0$, where $A_P$,$A_T$ are the mass numbers of the projectile and the target, respectively. The initial condition is $P(A_1, E_1, t = 0) = \delta_{A_1,A_P}$. In all cases investigated, the time step interval $\Delta\tau$ is taken to be from 0.05 to $0.1 \times 10^{-22}$sec. In the region where the kinetic energy loss increases faster, the transition probability changes also rapidly, the $\Delta\tau$ should be smaller. Throughout the evolution process the normalization of the distribution function must be preserved at the condition of the decay rate $\Lambda^{qf}_{A_1,E_1,t}(\Theta)$ being equal to zero.

The Master equation is coupled with the relative motion that the excitation energy $E_k$ and the interaction time $\tau_{int}$ (The evolution time t is from $t = 0$ to $t = \tau_{int}$) are calculated

by the parameterization method of the classical deflection function [21, 38] for each incident orbital angular momentum.

## III. RESULTS AND DISCUSSION

### A. The driving potential of the DNS and the optimal excitation energy to form SHN

From Eqs.(9-15), one finds that the nucleon transition probability in Eq.(3) is related to the size of the valance space $\Delta\varepsilon_k$ of Eq.(11), and so is related to the local excitation energy $\varepsilon^*$, which is a function of the mass asymmetry of the system via the driving potential of Eq.(17) for a certain angular momentum. Thus the driving potential is of vital importance for the dynamical diffusion process. The calculated driving potentials with and without considering the ground state deformations of nuclei for the system $^{70}Zn +^{208} Pb \rightarrow^{278}$ 112 are shown in Fig.1 as a function of the mass asymmetry variable $\eta$ in a bold solid line and thin dashed line, respectively. In the figure the ground state deformation $\beta_2$ of the nuclei of the DNS is taken from Ref. [39]. The bigger difference between the two lines indicates the bigger deformation of nucleus. The orientation of the deformed nuclei and the distance between the centers of the two nuclei are taken in a way which gives the lowest nucleus-nucleus interaction energy. Since the distribution function $P(A_1, E_1, t)$ in Eq.(3) is considered to cover the region from $A_1 = 0$ to $A_1 = A_P + A_T = A$, the driving potential has been calculated to cover $\eta = -1$ to 1. In Fig.1 the arrow at $\eta_i$ points to the incident channel. One nucleon transfer from $\eta_i$ to both sides, whether it is a neutron or a proton, depends in which direction the potential energy is lower. It turns out that the isotopic composition of the nuclei forming the DNS determined in this way does not deviate much from that following the condition of $N/Z$ equilibrium in the system. Consequently, the driving potential of Eq.(17) is an explicit function of neutron and proton numbers of fragments. In order to form a compound nucleus, a barrier $B^*_{fus}$ shown in the figure must be overcome. It is indicated that the deformation of the nuclei decreases the potential energy and the inner fusion barrier a great deal. The energy needed to pass over the barrier must be supplied by the incident energy. The survival probability demands the lowest excitation energy, so the optimal excitation energy of the compound nucleus indicated in the figure is $E^*_{CN} = U(\eta_i) + B^*_{fus}$ where $U(\eta_i)$ is the potential energy

of the initial DNS. For a set of cold fusion reactions, the driving potentials are calculated and the obtained optimal excitation energies of the compound nuclei from reactions based on Pb target are shown in Fig.2 in open circles, and compared with experimental data [6, 9] shown in solid dots. Good agreement is found.

### B. The fusion probability $P_{CN}$

Solving the Master equation Eq.(3) numerically, the time evolution of $P(A_k, E_k, t)$ to find fragment k (mass number $A_k$) with excitation energy $E_k$ at time t is obtained. All the components on the left side of the fusion barrier in Fig.1 contribute to the compound nuclear formation. The fusion probability $P_{CN}$ is the summation from $A_1 = 0$ to $A_{BG}$:

$$P_{CN}(J) = \int_{A_1=0}^{A_{BG}} P(A_1, E_k(J), \tau_{int}(J)) dA_1. \tag{21}$$

The intrinsic excitation energy $E_k$ is attributed to the kinetic energy loss of the relative motion. The calculation of the average energy loss, angular momentum loss, and interaction time has been described in detail in Ref. [21]. Here the relaxation times $\tau_R$ for radial kinetic energy, $\tau_{\mathcal{J}}$ for angular momentum, and $\tau_\epsilon$ for spheroidal deformation have been determined as $\tau_R \simeq 0.3 \times 10^{-21} s$, $\tau_{\mathcal{J}} \simeq 1.5 \times 10^{-21} s$ and $\tau_\epsilon \simeq 4 \times 10^{-21} s$. We plot the dissipated kinetic energy and the mean interaction time as a function of incident angular momentum J for $^{70}Zn +^{208} Pb$ with an optimal kinetic energy $E_{cm} = E^*_{CN}(10.29MeV) - Q = 252.37MeV$ in Fig.3. The large energy damping below the interaction barrier reflects the fragment deformation. The reversible shape oscillations or other coherent modes of excitation are not considered, only the effects of the deformations which become irreversible due to the coupling with the intrinsic degrees of freedom are taken into account [40]. The radial kinetic energy, the relative and the intrinsic angular momentum, the Coulomb interaction are all affected by the nuclear deformations, and also the interaction time $\tau_{int}(J)$. For partial waves with small incident angular momentum , the interaction time of the composite system is very long, and during this time a large amount of kinetic energy are dissipated and many nucleons are exchanged, some fraction of the distribution probability contributes to compound nuclear formation. The angular momentum dependence of $P_{CN}(J)$ for the above mentioned case is shown in Fig.4(a), where the angular momentum is cut off at about $20\hbar$ because at larger angular momentum the fission barrier for the compound nucleus becomes very small. About

95% fusion probability remain at $J = 10\hbar$, and about 68% at $J = 20\hbar$ with respect to $J = 0$. The dissipated energy, so as the excitation energy of nuclei are not influenced much by the incident angular momentum up to $J = 20\hbar$ as indicated in Fig.3(a). But it may be found in Fig.3(b) that the interaction time decreases rapidly with the increasing angular momentum. So the fusion probability $P_{CN}(J)$ decreases with the angular momentum slowly.

Fig.5(a) shows the calculated values of $P_{CN}$ for Pb-based reactions at nearly central collisions ($J \sim 0$) and with the reaction energies according to those optimal excitation energies indicated in Fig.2, respectively. Full dots are calculated results by Eq.(3) without considering the quasi fission. Open triangles are those including the quasi fission. One may find that $P_{CN}$ with the consideration of the quasi fission decreases by about four orders of magnitude with Z increasing from 106 to 118. Because the inner fusion barrier $B^*_{fus}$ increases with decreasing mass asymmetry of the initial DNS, i.e. with increasing Z for the Pb-based reactions, the fusion probabilities decrease rapidly with increasing Z. The straight line in the figure is used to guide the eye. The consideration of the quasi fission process in the master equation diminishes the fusion probability by one order of magnitude for $Kr + Pb \rightarrow 118$. The decreasing magnitude of the fusion probability becomes less and less for increasing asymmetry of the incident reaction system, since the inner fusion barrier is getting decreasing, and the distribution probability gets less chance to go to mass symmetrical direction, to which the quasi fission barrier is getting smaller.

### C. The survival probability of excited compound nucleus

The super-heavy compound nuclei are formed in excited states, and will lose excitation energy mainly by emission of particles and $\gamma$ quanta, and by fission. The surviving probability in cold fusion reactions estimates the competition between fission and neutron evaporation in the excited compound nucleus by a statistical model. In this case the width for the emission of a charged particle is much less than that for the emission of a neutron, and the $\gamma$ ray emission is important only when the excitation energy is smaller than the one-neutron separation energy.

In cold fusion reactions, the survival probability under one-neutron emission can be writ-

ten as:

$$W_{sur}(E^*_{CN}, J) = P_1(E^*_{CN}, J)\frac{\Gamma_n(E^*_{CN}, J)}{\Gamma_n(E^*_{CN}, J) + \Gamma_f(E^*_{CN}, J)}, \tag{22}$$

where $E^*_{CN}$, and $J$ are the excitation energy and the angular momentum of the compound nucleus, respectively. $P_1(E^*_{CN}, J)$ is the realization probability of the 1n channel at given $E^*_{CN}$ and $J$, which is calculated with the expression of Eq.(7) from Ref. [41]. $\Gamma_n$ and $\Gamma_f$ are the widths of neutron emission and fission, respectively. In calculating $W_{sur}$ the following formulae are used:

$$\Gamma_n(E^*) = \frac{1}{2\pi\rho(E^*)} \cdot \frac{2M_nR^2}{\hbar^2} g \int_0^{E^*-B_n-1/a} \varepsilon\rho(E^* - B_n - \varepsilon)d\varepsilon, \tag{23}$$

and

$$\Gamma_f(E^*) = \frac{1}{2\pi\rho(E^*)} \int_0^{E^*-B_f-1/a} \rho(E^* - B_f - \varepsilon)d\varepsilon, \tag{24}$$

where $\rho(E^*) = \frac{1}{\sqrt{48}E^*} exp[2\sqrt{aE^*}]$ is the level density, R ,$B_f$, $B_n$ are the radius, the fission barrier and the neutron separation energy of the compound nucleus, respectively. $M_n$ is the mass of the neutron, $g$ the spin factor of neutron and $a$ the level density parameter which is taken to be $a = A/12$. $E^*$ is the effective excitation energy of the compound nucleus. In Ref. [41] the fission barrier for SHE is divided into the macroscopic part $B_f^{LD}$, determined by liquid-drop model, and into the microscopic part $B_f^{Mic}$, determined by shell correction. The microscopic energy will be damped due to the dependence of the shell effects on the nuclear excitation. Thus, the fission barrier can be written as :

$$B_f = B_f^{LD} + B_f^{Mic}(E^* = 0)exp[-\frac{E^*}{E_D}] - (\frac{\hbar^2}{2J_{g.s.}} - \frac{\hbar^2}{2J_{s.d.}})J(J+1), \tag{25}$$

where $E_D$ is a damping factor describing the decrease of the influence of the shell effects on the level density with the increasing excitation energy of the nucleus, which is taken as

$$E_D = 0.4A^{4/3}/a, \tag{26}$$

where $A$ is the mass number of the nucleus. $J_{g.s.;s.d.} = k\frac{2}{5}MR^2(1 + \beta_2^{g.s.;s.d.}/3)$ are the moment of inertia of the fissioning nucleus at its ground state and the saddle state, respectively. Where $k \approx 0.4$ [42]. Since there are no data available, the quadrupole deformation parameters $\beta_2$ at the saddle point are taken from the microscopic calculation of the relativistic mean

field (RMF) theory [43], which has been proven to be quite successful for the description for exotic nuclei and superheavy nuclei (SHN) [44, 45, 46]. The angular momentum dependence of $W_{sur}$ for mentioned reaction $^{70}Zn+^{208}Pb \rightarrow^{278}$ 112 is indicated in the Fig.4(b). One may find that at $J \sim 15\hbar$ the survival probability decreases about one order of magnitude.

The macroscopic fission barrier can be evaluated by liquid drop model [47]. Taking the neutron separation energy and $B_f^{Mic}$ from Ref. [10], the calculated survival probabilities for one-neutron emission Pb-based reactions at nearly central collisions with the excitation energies from Fig.2 are shown in Fig.5(b). The tendency of the results is basically consistent with that shown in Fig.4 of Ref. [41].

### D. The evaporation residue cross section

Applying Eqs.(1) and (2) we calculated the evaporation residue cross sections. For cold fusion reactions with the optimal excitation energies as indicated in Fig.2, a set of evaporation residue cross sections for Pb-based reactions are shown in Fig.6 with solid stars, the solid dots are experimental data quoted in Ref. [48], and some estimated data for element 114, 116, and 118 by different groups are indicated in the figure [49, 50, 51]. The upper limit for element 118 was estimated by LBNL recently [52] and also shown in this figure. Our results are in principle in agreement with the data within one order of magnitude.

## IV. SUMMARY

The fusion probability is calculated in very strongly damped reaction processes, where large amounts of the relative kinetic energy are changed into intrinsic excitation energy and nucleons transferred from the lighter fragment to the heavier one to produce super-heavy nuclei in the tail of the heavy-fragment mass distribution. Within the DNS conception, instead of solving FPE analytically, the Master equation is solved numerically in order to calculate the fusion probability, so that the harmonic oscillator approximation to the potential energy of the DNS, which is the very entrance of the nuclear structure of the model [15, 16], is avoided. In our calculations the relative motion including the relaxations of the energy, angular momentum, and fragment deformation is explicitly treated, so that the nucleon transition probabilities, which are derived microscopically, are coupled with

the relative motion and thus are time-dependent. Comparing with the analytical (or the logistic-type) solution of FPE, our results preserve more dynamical effects. The fusion process is calculated for each partial wave, and an about 32% fusion probability reduction is found for the $J = 20$ partial wave compared with that for the central collision to form compound nucleus 112. And the survival probability at $J \sim 15\hbar$ decreases about one order of magnitude. Our calculated evaporation residue cross sections for one-neutron emission channel of Pb-based reactions are basically in agreement with the experimental data within one order of magnitude. However, although the driving potential has been calculated in $\eta$ and R two dimensions, the diffusion process to the two dimensions are not treated simultaneously. The quasi fission is treated by a decay rate of Kramers' type. The Master equation should be extended into a two dimensional case by taking the distance between the centers of nuclei into account in addition, so that the qusi fission could be described in the process to fully understand the reaction dynamics. Presently, the nucleus-nucleus interaction with deformations is only simulated by shifting the distance between surface of spherical nuclei to a smaller relative distance determined by the same distance as those which the deformed nuclei have, and in this way it has been a little overestimated, especially in the region where nucleus has bigger deformation. Therefore, the nucleus-nucleus interaction including the consideration of nuclear deformation is being investigated by using the Skyrme-type force, which we will include in the calculation later, and would like to publish elsewhere [53]. In future a time-dependent multidimensional potential energy surface has to be built up, and to treat the time-dependent dynamics in order to get a complete quantitative understanding of the fusion reaction mechanism of heavy nuclei.

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## Figure Captions:

Fig.1: The driving potential of the DNS for the system $^{70}Zn +^{208} Pb \rightarrow^{278}$ 112 as a function of the mass asymmetry variable $\eta$. $BG$ marks the top point of the potential energy.

Fig2: The optimal excitation energies of the compound nuclei from reactions based on Pb target as a function of the charge number of compound nuclei. The calculated results are shown with open circles, and the experimental data with solid dots.

Fig3: The mean dissipated kinetic energy and the mean interaction time of the relative motion are shown as a function of the incident angular momentum J in (a) and (b), respectively, for $^{70}Zn +^{208} Pb$ reaction with the corresponding optimal excitation energy $E^*_{CN} = 10.29MeV$ of the compound nucleus $^{278}$112.

Fig4: (a): The angular momentum dependence of $P_{CN}(J)$ for the same case as in Fig.3. (b): The corresponding angular momentum dependence of $W_{sur}$.

Fig5: (a): the calculated values of the fusion probability $P_{CN}$ for one-neutron emission Pb-based reactions at nearly central collisions and with the reaction energies according to those indicated in Fig.2 as a function of the charge number of the compound nuclei. The open triangles and solid dots stand for the fusion probability $P_{CN}$ with and without considering the effect of the quasi fission, respectively. The corresponding mass number are listed on the second row. (b): The corresponding calculated survival probability.

Fig6: The evaporation residue cross sections for one-neutron emission Pb-based reactions with the excitation energies from Fig.2 as a function of the charge number of compound nuclei. Our calculated results are indicated by solid stars, the experimental data by solid dots. And some estimated data for element 114, 116, and 118 by different groups are indicated with different symbols.